# Urban Economics and Simulations

Slides A

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# Outline

# Overview

# Slides B

Content:

- Game theory
- Transport II
- Hotelling game

# Topics

Some questions we are interested in are:

- Where are factories localized?
- How are cities spatially organized?
- Where do people live in the city?
- In which city do people want to live?
- How to analyse commuting traffic?
- How to handle congestion?

# Microeconomic models

The first four questions are dealt with in Chapters 1, 2, 3 and 8 of the Text Book of Brueckner.

Questions 5 and 6 are dealt with in its Chapter 5. Besides the approach in Chapter 5, we shall present a more powerful one in Slides B. In order to do so, we need to have a quick look to some game theoretic concepts (which also may be useful elsewhere as we shall see).

# Microeconomic models (ctd.)

We shall approach these (interrelated) questions from an economic perspective. This we do by using relatively simple little neoclassical microeconomic models.
I will go a little bit beyond the mainly graphical analysis in the Text Book (Do not worry.) In this context i like to mention:

> La géométrie est l'art de raisonner juste sur des figures fausses. (René Descartes)

Making one coherent big model, still has not been realized!

A prominent role in these models is played by transport costs for inputs and outputs and commuting costs for residents. This also explains that there is special attention to the transport topic in this course.

# Overview

# Short answer

Why do cities exist?

Short answer: from an economic point of view a short answer is that jobs (offered by factories) are concentrated that in turn leads to a concentration of residences as people like to locate near their work sites. The result is a city. Of course there are other points of view! (Military historian: defence against attack. Sociologist: interact socially.)

So it is important to have insight into the location problem for factories . Two forces are important in this context:

- Scale economies.
- Agglomeration economies.

# Scale economies

Scale economies concern the specific cost advantages that factories may obtain due to their scale of operation (typically measured by the amount of output produced):

Cost per unit of output (i.e. average costs) is a decreasing function of output .

One important reason for scale economies is that the production function exhibits increasing returns to scale. Let us quickly reconsider this important microeconomic notion.

# Intermezzo: increasing returns to scale

The fundamental object here is that of a production function for a firm (factory).

Assume (for simplicity) two production factors 1 and 2 (may be labour and capital) and consider a production function $f(k_1, k_2)$ for the firm. One refers to $(k_1, k_2)$ as input and to $q = f(k_1, k_2)$ as output.

One says that the production function $f$ exhibits

- increasing returns to scale if $f(\lambda k_1, \lambda k_2) > \lambda f(k_1, k_2)$ for all $(k_1, k_2)$ and $\lambda > 1$;
- constant returns to scale if $f(\lambda k_1, \lambda k_2) = \lambda f(k_1, k_2)$ for all $(k_1, k_2)$ and $\lambda > 1$;
- decreasing returns to scale if $f(\lambda k_1, \lambda k_2) < \lambda f(k_1, k_2)$ for all $(k_1, k_2)$ and $\lambda > 1$.

# Intermezzo: increasing returns to scale (ctd.)

Suppose that the price of one unit of production factor $i$ is $w_i$.

Cost minimisation problem

$$\underset{\substack{k_1,k_2\\ f(k_1,k_2)=q}}{\text{MIN}} \quad w_1 k_1 + w_2 k_2.$$

Optimal $k_1$ and $k_2$, denoted by $k_1^\star$ and $k_2^\star$, depend on $q$ (and $w_1$ and $w_2$); the so-called conditional production factor demand functions .

- Cost function $C(q) = w_1 k_1^\star(q) + w_2 k_2^\star(q)$.
- Average cost function $AC(q) = C(q)/q$.
- Marginal cost function $MC(q) = C'(q)$.

# Intermezzo: increasing returns to scale (ctd.)

The important result for the average cost function $\mathrm{AC}$ is:

increasing returns to scale $\Rightarrow$ $\mathrm{AC}$ is *decreasing*;

constant returns to scale $\Rightarrow$ $\mathrm{AC}$ is *constant*;

decreasing returns to scale $\Rightarrow$ $\mathrm{AC}$ is *increasing*.

Exercise 1 (in the file “Exercises A”) is devoted to these statements for the simple case with 1 production factor. There it is quite simple to determine the conditional production factor function. Later we shall explain how for two production factors $k_1^\star$ and $k_2^\star$ can be determined. (For the moment not needed.)

# Agglomeration economies

So scale economies favour the formation of large firms and therefore can generate a city. However, for really large cities, many firms must locate in close proximity. This brings us to the other force: agglomeration economies.

Agglomeration economies concerns the fact that a firm benefits from locating amid other ones.

Whereas scale economies operate within a firm, without regard to the external environment, agglomeration economies are external to the firm.

# Agglomeration economies (ctd.)

The notion of "agglomeration economies" is more difficult to grasp (formalize) than that of "scale economies".

There are two types of agglomeration economies:

- technological agglomeration economies : raised labour productivity by knowledge spillovers and socializing of workers.
- pecuniary agglomeration economies : reduce cost of inputs without affecting labour productivity, for example by hiring specialised labour. However, most importantly this concerns saving on transportation costs when a firm locates in a city that contains its input suppliers and its market.

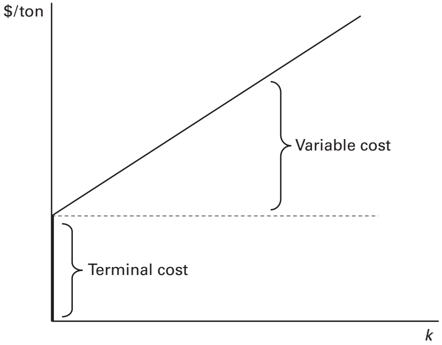
Next we shall pay special attention to transport costs for firms.

# Transport costs

- Transport costs : the expenses involved in moving products or assets to a different place, which are often passed on to consumers.
- Transport costs $C$ in any case depend on weight $q$ (may be volume) and distance $k$. We write $C(q, k)$.
- If there is proportional dependence of transport costs on weight, so $C(q, k) = C(1, k)q$, then it make sense to speak about transport costs per ton $C(1, k)$. Note that proportional dependence implies that $C(0, k) = 0$. Further we always suppose this situation.
- Transport costs per ton come as terminal i.e. $C(1, 0)$, and operating costs, i.e. $C(1, k) - C(1, 0)$. So terminal (operating) costs are fixed (variable) costs. Fixed costs are relatively low for trucks and relatively high for trains. (This makes that trucks are interesting for short-distance transport and trains for long-distance transport.)

# Transport costs (ctd.)

Now let us look to the dependence of costs on distance. Fixed costs make that average transport costs per kilometer often exhibit economies of distance in the sense that they are a decreasing function of distance.



# Transport costs (ctd.)

- For us the distinction between transport costs for input and for output is important.
- Weight-losing industries : weight of output is less than weight of input. Examples: mining, sugar industry, ... .
  Weight-gaining industries : weight (may be volume) of output is greater than weight of input. Examples: soft drinks, bread ... .
- The distinction in the previous bullet makes that it makes sense to (re-)define transport costs for input as as costs for enough input to produce one ton of output. This we shall do now always. (If one is not doing this, then one has to take into account the weight loss or gain when calculating costs.)

# Intermezzo: convexity and concavity

Convexity and concavity is the most important mathematical structure for economists.

Do You know why?

(By the way: do You know what we mean by a concave function?)

Three nice properties of a concave function $f : I \to \mathbb{R}$ with $I \subseteq \mathbb{R}_+$ and $f \geq 0$:

- if $f$ is differentiable and $f'(x) = 0$, then $x$ is a maximiser;
- the average function $\overline{f}$ is decreasing;
- (Bauers theorem): the minimisers of $f$ with domain $I$ a segment (i.e. closed interval) are at an endpoint.

# Best location for a factory

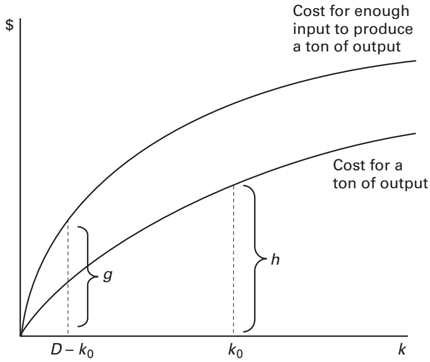
Consider the problem of minimizing total transport costs for transporting over a given distance an input from a mine to a factory and for transporting the output to a market. We assume an 1-dimensional situation. The optimal location is the location of the factory where the sum of the transport costs of a given input and to this input belonging output is minimal.

Distance between market and mine is $D$ km:


If the factory is located at distance $k_0$ km from the market, then the input must be shipped $D - k_0$ km and the output $k_0$ km.

# Best location for a factory (ctd.)

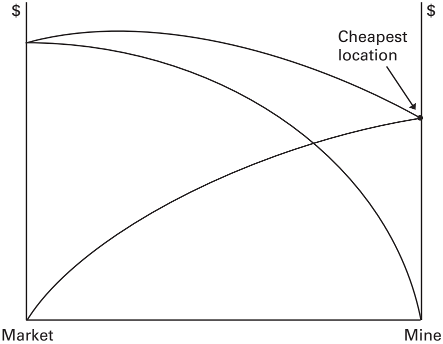
Consider the situation the next figure. So the industry is weight-losing, there are no terminal costs and there are economies of distance.




Solution: optimal $k_0$ such that total costs $h + g$ is minimal.

# Best location for a factory (ctd.)

Redraw figure:




Lower curve concerns output.

(WARNING: This figure is taken from the Text Book and is not completely correct.)

# Best location for a factory (ctd.)

Here: optimal location at the mine. So only the output should be shipped.

Conclusion: suppose cost curves of input and output are concave.

- If industry is weight-losing, then the optimal location for the firm is at the mine.
- If industry is weight-gaining, then the optimal location for the firm is at the market.

# Best location for a factory (ctd.)

These (general) results, depend on the concavity of the cost curves of input and output which makes that also their sum is concave and thus Bauers theorem applies.

In Exercise 3 (in the file “Exercises A”) , this result will be illustrated with a numerical example.

However, in cases where the shipment must be loaded and reloaded for some reason, the best location may lie at a transshipment point between the mine and the market. (Exercise 1.1 in the Text Book concerns an example.)

# Location of factory

- Production (scale economies); transport (economies of distance).
- In order to deal with optimal location of a factory not only economies of distance matter, as we considerd above, but also scale economies. This may be a complex problem.
- Also a question is whether to centralize production or to divide among a number of smaller establishments may give a better solution.
- In Exercise 4 (in the file "Exercises A") we shall consider a concrete simple example that clarifies the involved problems.

# Final remarks

The material in this section relates to the so-called New Economic Geography , an area of active research since the early 1990s.

Further reading:
E. Glaeser and J. Kohlhase; Cities, regions and the decline of transport costs, Papers in Regional Science, 83, 197-228, 2004.

# Overview

# Basic assumptions

Consider a city with the following structure: a CBD (Central Business District), collapsed to one point, and city neighbourhoods. Further there are single-person households (referred to as residents) living in this city.

A. All jobs are in the CBD.

B. The city has a network of radial roads leading to the CBD.

C. Each resident rents a dwelling.

D. Residents are spatially uniformly distributed over the city: denote with $x$ the radial distance from a specific resident to the CBD.

E. Goal of residents is to maximise utility.

F. Each resident consumes housing and another (composite) good, called “bread”, with price 1. Housing is assumed to be an ordinary good.

# Main questions

Interesting questions are: how depend

- rental price of dwellings
- consumption (housing and bread),
- land rent,
- building heights,
- population density

on the distance $x$ to the CBD?

We are going to build a model in order to study these questions. The model focuses on commuting costs. We start with analysing the consumer side (giving an answer to the first 2 questions) and then later the producer side (answering the last 3).

# Additional assumptions

But, in order to do so we make the following additional assumptions which later will be relaxed.

G. All residents are identical.

H. All residents use the same transport mode to get to work.

# Quantities

Given a resident with radial distance $x$ to the CBD. Let

- $q$ floor space in square meter in the dwelling;
- $c$ amount of bread per consumed unit;
- $p$ rental price in euro per square meter.

Parameters (the same for all residents):

- $t$ commuting-cost per km ('out-of-pocket') cost in euro ;
- $y$ income in euro;
- $u(q, c)$ utility function of resident.

# Dependence on distance

Optimal floor space $q$, amount of bread $c$ and rental price depend on the distance $x$:

$$q^\star(x),$$

$$c^\star(x),$$

$$p^\star(x) \quad \text{housing price curve.}$$

Our goal is to find out how.

# Intermezzo: utility maximisation

Consumer, consuming two goods, with ordinal utility function $u(x_1, x_2)$, prices $p_1, p_2$ and income $m$.

Utility maximisation problem :

$$\text{MAX}_{\substack{x_1, x_2, \\ p_1 x_1 + p_2 x_2 = m}} u(x_1, x_2).$$

Optimal $x_1^\star$ and $x_2^\star$ depend on $p_1, p_2$ and $m$. These functions are called Marshallian demand functions .

# Intermezzo: utility maximisation (ctd.)

Optimal $x_1^\star$ and $x_2^\star$ often can be determined by solving

$$p_1 x_1 + p_2 x_2 = m \quad (\text{ budget restriction (line) }),$$

$$p_1/p_2 = \frac{\partial u}{\partial x_1}\Big/\frac{\partial u}{\partial x_2} \quad (\text{ Second law of Gossen }).$$

Note: with $x_1$ on horizontal axis and $x_2$ on vertical axis, the slope of the budget line is $-p_1/p_2$.

Graphically: in the optimum $(x_1^\star, x_2^\star)$, the indifference curve through this bundle is tangent to the budget line.

# Intermezzo: utility maximisation (ctd.)

One distinguishes among four types of goods.
Good $i$ is

- Giffen if $p_i \uparrow \Rightarrow x_i^* \uparrow$.
- ordinary if $p_i \uparrow \Rightarrow x_i^* \downarrow$.
- normal if $m \uparrow \Rightarrow x_i^* \uparrow$.
- inferior if $m \uparrow \Rightarrow x_i^* \downarrow$.

# Intermezzo: utility maximization (ctd.)

For example, for the Cobb-Douglas utility function

$$u(x_1, x_2) = x_1^{\alpha_1} x_2^{\alpha_2}$$

one has the formulas

$$x_1^\star = \frac{\alpha_1}{\alpha_1 + \alpha_2} \frac{m}{p_1}, \quad x_2^\star = \frac{\alpha_2}{\alpha_1 + \alpha_2} \frac{m}{p_2}.$$

# Analysis

Commuting costs: $tx$

$y - tx$: disposable income for consumption.

Budget constraint (line):

$$pq + c = y - tx.$$

More explicitly: $p^\star(x)q^\star(x) + c^\star(x) = y - tx$.

Each resident, characterized by $x$, wants, given his disposable income, to choose $q$ and $c$ such that utility is maximal.

# Example

Utility function $u(q, c) = qc$.

Optimal $q$ and $c$:

$$q^{\star}(x) = \frac{1}{2}\frac{y - tx}{p^{\star}(x)}, \quad c^{\star}(x) = \frac{1}{2}\frac{y - tx}{1}.$$

So in this example, and also in the general problem, the following problem remains:

What is $p^{\star}(x)$?

How to solve this problem?

# Intermezzo: optimization and equilibrium principle

Well, in economic theory the following two principles play an important role:

- optimisation principle (utility maximisation, cost minimisation, profit maximisation, ...).
- equilibrium principle (supply equals demand, nobody regrets his choice, ...)

Often both principles are necessary for a full analysis. Most important example from microeconomics is the general equilibrium theory.

# Optimisation and equilibrium principle (ctd.)

In our problem:

Optimization principle: utility maximization of residents.

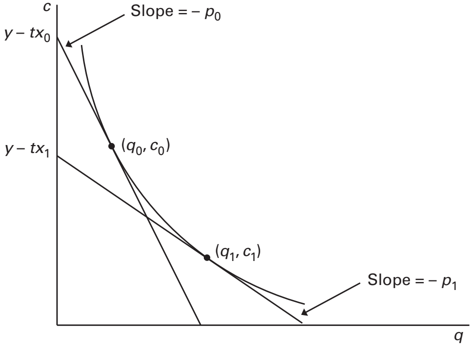
Equilibrium principle: in equilibrium (assuming this exists) each resident is equally well of at all locations.

In addition, as residents are, by Assumption G, identical (i.e. same utility function): in equilibrium (assuming this is unique) each resident achieves the same utility.

Therefore $u(q^\star(x), c^\star(x))$ is independent of $x$. This fact will be used in the following graphical analysis.

# Graphical analysis

Consider a central city-resident located at $x_0$ and a suburban resident located at $x_1$. So $x_0 < x_1$.




We see: $p_0 > p_1$ (as $-p_1 > -p_0$), $q_0 < q_1$ and $c_0 > c_1$.

# Main results

rental price $p$ is a decreasing function of $x$.

floor space $q$ is an increasing function of $x$.

bread $c$ is a decreasing function of $x$.

# Intuition

Since higher commuting costs mean that disposable income falls as $x$ increases, some offsetting benefit must be present to keep utility from falling. The offsetting benefit is a lower price per square meter of housing at greater distances.
As housing is an ordinary good (i.e. not a Giffen good), the amount of floor space is an increasing function of $x$.

# Housing price curve

With a little bit microeconomics and mathematics, one can prove for the housing price curve $p^\star(x)$ the formula

$$\frac{\partial p^\star}{\partial x}(x) = -\frac{t}{q^\star(x)}.$$

This result will play later on a very important role.

Note: the behaviour of total rent , i.e. $p^\star(x)q^\star(x)$, on $x$ is ambiguous. So the total rent for a suburban dwelling can be either larger or smaller than the total rent for a central-city one.

# Basic assumptions

In addition to the previous assumptions A–H of the model consider housing developers who build buildings, divide the buildings into dwellings and rent the dwellings to residents.

I. The goal of the housing developers is profit maximisation.
J. Housing developers are willing to build housing in all locations.
K. There is no open space between buildings.

# Main questions

Denoting (again) with $x$ the radial distance from a building to the CBD the remaining questions are the dependence on $x$ of

- land rent,
- building heights,
- population density.

# Additional assumptions

L. All housing developers are identical.

M. Each building is produced only with land and capital (i.e. building materials). (So labor is not considered.)

N. Production: constant returns to scale.

# Quantities

Given a housing developer with building at distance $x$

- $Q$ amount of floor space in building .
- $N$ amount of capital .
- $l$ amount of land .
- $r$ land rent .

Note that $N/l$ is building height (in fact is only proportional to).

Parameters (the same for all housing developers):

- $i$ price of capital .
- $H(l, N)$ production function of housing developer.

# Intermezzo: profit maximisation

Symbols: $f(k_1, k_2)$ production function, $w_1, w_2$ prices of production factors and $p$ output price.

Profit function:

$$\pi(k_1, k_2) = pf(k_1, k_2) - (w_1 k_1 + w_2 k_2).$$

Profit maximisation implies cost minimisation.

Important remark: if there are constant returns to scale and the producer has a profit maximising input $(k_1, k_2) \neq (0, 0)$ (i.e. the producer is active), then the maximal profit is 0 and prices are such that “price is marginal costs” hold, i.e. such that

$$p = \mathrm{MC}(q).$$

# Intermezzo: profit maximisation (ctd.)

Illustration of correctness of last statement for the case of one production factor.

$$\pi(k) = pf(k) - wk.$$

Constant returns to scale of $f$ implies $f(k) = \beta k$ for some $\beta$, thus

$$\pi(k) = p\beta k - wk = (p\beta - w)k.$$

We see:

$p\beta = w \;\Rightarrow\;$ maximal profit is 0 and firm can be active. (Here $p = w/\beta$, i.e. price equals marginal costs.)

$p\beta < w \;\Rightarrow\;$ maximal profit is 0 and firm is not active.

$p\beta > w \;\Rightarrow\;$ maximal profit does not exist.

# Analysis

$Q = H(l, N)$.

Constant returns to scale: $H(\lambda l, \lambda N) = \lambda H(l, N)$.

Costs: $rl + iN$. Slope of isocost line is $-r/i$.

Profit: $pH(l, N) - (rl + iN)$. More precisely

$$p(x)H(l(x), N(x)) - \Big(r(x)l(x) + iN(x)\Big).$$

# Analysis

A correct analysis of the behaviour of the housing developers is much more difficult than that of the residents!

Let us start with the equilibrium principle. By Assumption J, housing developers are willing to build housing in all locations. This implies that there is a spatially uniform profit. (As there is constant returns to scale, there even is a “normal economic profit”: profit 0.)

But as the housing price curve $p(x)$ is decreasing, profits will not be the same unless a compensating differential exists on the cost side: as the price of capital $i$ is fixed, land rent $r$ must be lower in suburbs than at central locations. (One can make this reasoning more precise with some microeconomic theory.)

Conclusion: $r$ is a decreasing function of $x$ .

# Analysis (ctd.)

Each housing developer characterised by $x$, wants to maximise his profit. In order to do so he has to minimise costs given an amount of floor space $Q$.

# Intermezzo: Cost minimisation (ctd.)

Consider again the cost minimisation problem

$$\text{MIN}_{\substack{k_1, k_2 \\ f(k_1,k_2)=q}} \quad w_1 k_1 + w_2 k_2.$$

Optimal $k_1^\star$ and $k_2^\star$ often can be determined by solving

$$w_1/w_2 = \frac{\partial f}{\partial k_1} \Big/ \frac{\partial f}{\partial k_2},$$

$$f(k_1, k_2) = q.$$

Graphically: in optimum production factor bundle $(k_1^\star, k_2^\star)$, the isocost line through this bundle is tangent to the isoquant through this bundle.

# Graphical analysis

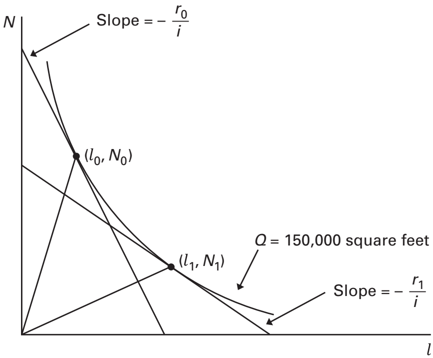
Consider the cost minimisation problem for producing a given amount of floor space $Q$ for a central city housing developer located $x_0$ and a suburban housing developer located at $x_1$.

So $x_0 < x_1$. We already know that

$$r_0 > r_1.$$

This implies for the slopes of the isocost lines at the optima $-r_1/i > -r_0/i$.

# Graphical analysis




We see $N_0/l_0 > N_1/l_1$.

# Graphical analysis (ctd.)

Conclusion: building height $h \sim N/l$ is a decreasing function of $x$.

Thus we have:

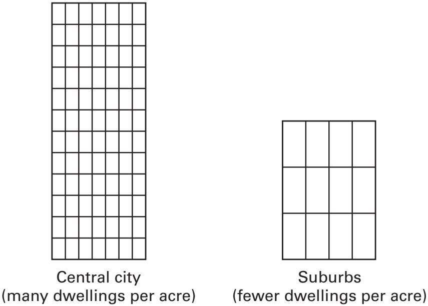
$$x_0 < x_1,$$

$$q_0 < q_1,$$

$$h_0 \sim \frac{N_0}{l_0} > h_1 \sim \frac{N_1}{l_1}.$$

# Graphical analysis (ctd.)

By Assumption K, there is no open space between buildings. Also floor space $q$ is an increasing function of $x$. This implies the following situation:




Thus: population density $D$ is a decreasing function of $x$.

# Main results

Old:

Rental price $p$ is a decreasing function of $x$.

Floor space $q$ is an increasing function of $x$.

Bread $c$ is a decreasing function of $x$.

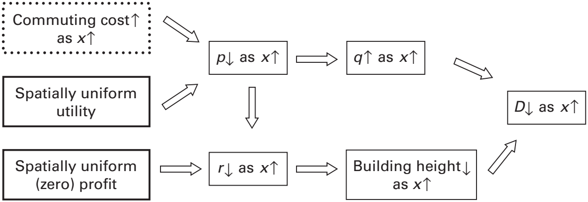
New:

Price of land $r$ is a decreasing function of $x$.

Building height $h$ is a decreasing function of $x$.

Population density $D$ is a decreasing function of $x$.

# Logical structure of the analysis

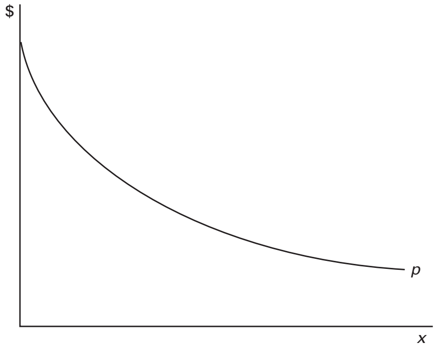


# House-pricing curve

Remember the formula

$$\frac{\partial p^\star}{\partial x} = -\frac{t}{q^\star(x)}.$$

As $q^\star$ is an increasing function of $x$, $\frac{\partial p^\star}{\partial x}$ is a decreasing function of $x$; therefore the house-pricing curve $p^\star(x)$ is convex.



# Comparative statics

How depend rental price $p$ and land rent $r$ on commuting cost parameter $t$ and income parameter $y$?

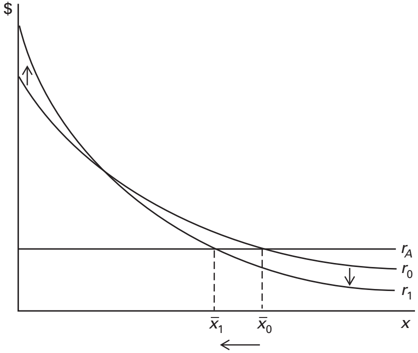
(Sophisticated) mathematical analysis gives

- An increase of $t$ leads to a clockwise rotation of the house-pricing curve $p(x)$ and the land rent curve $r(x)$.
- An increase of $y$ leads to a counterclockwise rotation of the house-pricing curve $p(x)$ and the land rent curve $r(x)$.

# Intuition

Suppose *t* increases. Suburban commuters will want to move towards the center to reduce their commuting costs. This movement bids up housing prices near the CBD, and reduces them at suburban locations. As a result, the housing-price curve rotates in a clockwise direction.

The profit of housing developers then rises near the center and falls in the suburbs. Land rents then rise near the center and fall in the suburbs, causing a clockwise rotation in the land-rent curve.




Effect of a higher $t$ on the land rent: clockwise rotation.

And: mathematical analysis shows that the housing price curve rotates in a counterclockwise direction if consumer income $y$ increases.

# Weaker assumptions

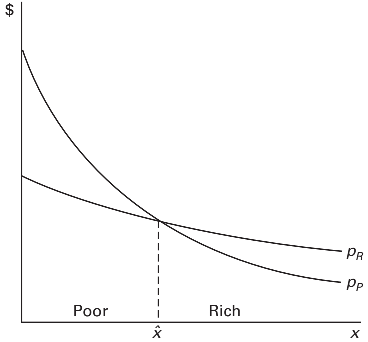
- Two income groups (rich and poor).
- Modifying the (radial) transportation system.
- ...

Remark: dealing with modifications of the transportation system will happen when we deal later with the Transport part of the course.

# Two income groups

Poor and rich: $y_P < y_R$.

Two housing price curves: $p_P$ and $p_R$. The “rotation result” leads to the following figure:



# Two income groups (ctd.)

This figure is qualitatively correct:

- At $\hat{x}$, the two groups face a common rental price $p$.
- At this price $q_R > q_P$.
- So $-t/q_P < -t/q_R$.
- Thus at $\hat{x}$ the slope of $p_P$ is less than the slope of $p_R$.

Nice reasoning!

# Two income groups (ctd.)

Only upper curve at each location is the new price housing curve: to actually reside at a particular location, members of a given income group must be the highest bidder.

Conclusion: Poor live in center, rich in suburbs.

However, if travelling time is taken into account, this result does not hold anymore.

# Overview

# Freeway congestion

The model in the Text Book (Chapter 5) deals with the topic of freeway congestion.

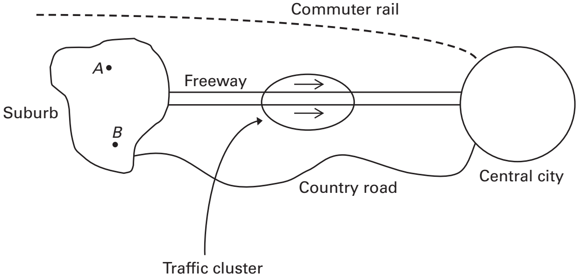
Freeway congestion is an example of a negative consumption externality: cars slow down each other. Although for each commuter congestion may cause a small cost (i.e. personal time lost), added up costs are non-negligible.

Now we shall have a look to a simple model that gives some first insight into the problems involved.

# Assumptions

Model relates to commuter trips on a single freeway sensible to congestion between a suburb and the central city. Besides the freeway there are some alternate routes that consumers can take without congestion.

Each commuter *a* has a preferred alternate route that is best in the sense that it has the lowest costs, say $g_a$, among alternatives to the freeway.



# Question

The question is what each commuter will do: choosing the freeway or the alternate route (i.e. commuter rail, country road, ...).

In order to answer this question, we make now the following model.

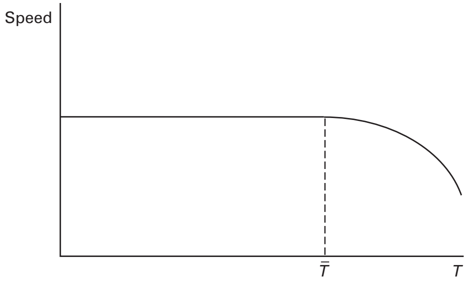
# Quantities

The ingredients of the model are the following objects:

- $T$ number of cars on freeway .
- $\overline{T}$ freeway capacity .
- $L$ lenght of freeway (in km).
- $s(T)$ traffic speed (in km / hour).
- $m$ money cost of a trip (in euro).
- $w$ wage (per hour).

# Quantities (ctd.)

Traffic speed $s(T)$



# Derived quantities

Time duration of trip: $\frac{L}{s(T)}$.

Cost of one trip: $g(T) = m + w\frac{L}{s(T)}$.

Aggregate costs: $C(T) = Tg(T)$.

Average aggregate costs: $AC(T) = g(T)$.

Marginal aggregate costs $MC(T) = g(T) + Tg'(T)$ (assuming $T$ is a continuous variable).

# Intermezzo: Principle of the marginal leads the average

If $f(x)$ is a function, for example a cost function or production function, then let

$$\bar{f}(x) := \frac{f(x)}{x} \text{(average function)},$$

$$f'(x) := \frac{df}{dx} \text{(marginal function)}.$$

The Principle of the marginal leads the average states:

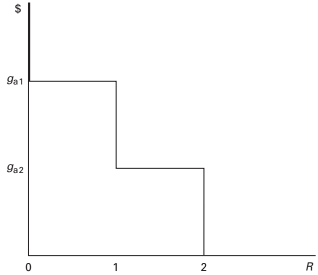
As long as the marginal function is below the average function, the average function decreases and as long as the marginal function is above the average function, the average function increases.

# Aggregate demand

Let $T(D)$ be the aggregate demand for using the highway, i.e. $T(D)$ is the number of commuters that use the highway if the cost for using it is $D$.

$T(D)$ can be derived from the costs of the best alternative routes of the commuters.

# Aggregate demand (ctd.)

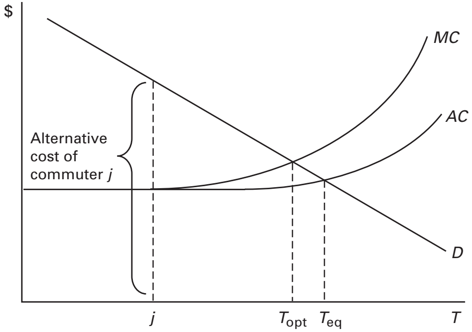



($R$ is number of freeway trips.)

If the commuters are $1, 2, 3, ...$ and such numbered that their alternate costs $g_{a_1}, g_{a_2}, g_{a_3}, \ldots$ are decreasing, then it can be shown that the height up to the demand curve at $T = j$ is equal to the alternate cost of commuter $j$. (See the book for the detailed reasoning that gives this result.)

# Results

Graphically, assuming many commuters.




$(D(T)$ is the inverse of $T(D))$

Exercise 7 (in the file “Exercises A”) deals with a simple numerical example.

# Results (ctd.)

The main results of the analysis can be summarized as follows (see the quite subtle reasoning in the Text Book):

- Equilibrium number $T_{eq}$ of commuters on freeway: solve $D(T) = AC(T)$.
- Social optimum number $T_{opt}$ of commuters on freeway: solve $D(T) = MC(T)$.
- $T_{eq} > T_{opt}$.

$T_{eq}$ is the number of commuters that use the freeway in equilibrium. Commuter 1 through $T_{eq}$ use the freeway, while commuters $T_{eq} + 1$ trough the total number of commuters use the alternate route.

# Results (ctd.)

And $T_{opt}$ is the number of commuters on freeway such that total cost of commuting (freeway users and alternate-route users) is minimal.

We see: equilibrium is not a social optimum. Reason: as $T_{opt} < T_{eq}$, there are too many cars on the freeway.

# Tolls

One may handle the congestion problem by imposing congestion tolls: each car must pay a toll equal to the "external costs" it generates. This makes that more commuters also choose alternate routes or travel at a different time.

By the way: as explained in Chapter 4 of the Text Book (which we skipped), congestion tolls in the urban model leads the residents to move to the CBD, which makes the city more compact.

For more on tolls (related to the Netherlands) see: T. Tillema, E. Ben-Elia, D. Ettema, J. van Delden, Charging versus rewarding: a comparison of road-pricing and rewarding peak avoidance in the Netherlands, Transport Policy 26, 4-14, 2013.

# Quality of model

Model is very interesting as one can quickly can address various problems. However, it makes heroic assumptions, its analysis (in fact) is subtle and contains some ad hoc reasonings.

Using a little bit of very elementary game theory, we now shall consider (in Slides B) a more general model. The nice thing is that the setup of the model is straightforward (but its analysis may not). It would be very interesting to reproduce with game theory the above result. This is one of the things Your teacher is working on!

# Reflection

There is much more going on: for example, have a look to the following video and then think about how the model should be extended/modified in order to deal with the issues in this video.

https://www.youtube.com/watch?v=iHzzSao6ypE

# Overview

- Short introduction
- Why do cities exist?
  - Increasing returns to scale
  - Transport costs
  - Location problems
- Urban spatial structure
  - Consumer analysis
  - Producer analysis
  - Modification of assumptions
- Transport I
  - Model in Text Book
- Local public goods and services
  - Public goods
  - Local public good

# 4 types of goods

A good (or service] is called rivalrous if its consumption by one subject reduces its consumption by another subject. A good is called excludable if its supplier can exclude subjects (possibly at a small cost) from consuming it. Combining these properties gives four types of goods:

| | rival | non-rival |
|---|---|---|
| excludable | private good | artificial scarce good |
| non-excludable | common resource | public good |

Economic theory spends a lot attention to private goods. Dealing with the other three types complicates the considerations.

# Real-world examples

The classic example of a public good is a lighthouse. Other examples are defence, (not coded) radio broadcasts and the spelling of the Dutch language.

An example of an artificial scarce good is a computer program.

An example of a common resource is fishing in public waters.

An example of a private good is a bottle of lemon-flavored vodka.

# Effects

From microeconomic theory one knows that under perfect competition, there are no market failures. However, if there are goods that are non-rival or non-excludable, this is usually no longer the case.

If a good is non-rival or non-excludable, then perfect competition leads to an inefficient (low or high) quantity of that good.

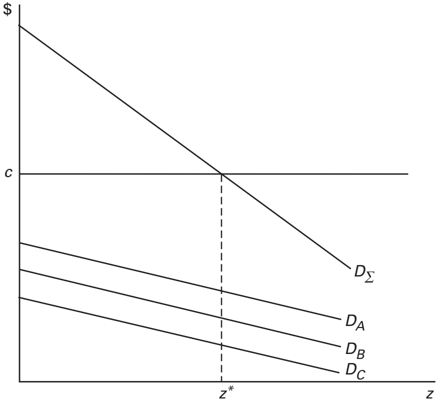
# Social optimal level

Denote the level of the public good by $z$ and its cost per unit be $c$. Denote the marginal benefit of a person $X$ by $D_X(z)$. It is a reasonable assumption that all $D_X(z)$ are strictly decreasing functions of $z$. Let $D_\Sigma(z) := \sum_X D_X(z)$.

The social optimal level of a public good is the amount $z^\star$ where the marginal social benefit (from an additional unit) equals its costs $c$, i.e. where $D_\Sigma(z^\star) = c$. (Do You understand why?)

This is illustrated with a stylized example in the Text Book: the public good is police protection and there are three persons: $A$, $B$ and $C$. The situation there is represented in the following figure.

# Social optimal level (ctd.)



# Notion of local public good

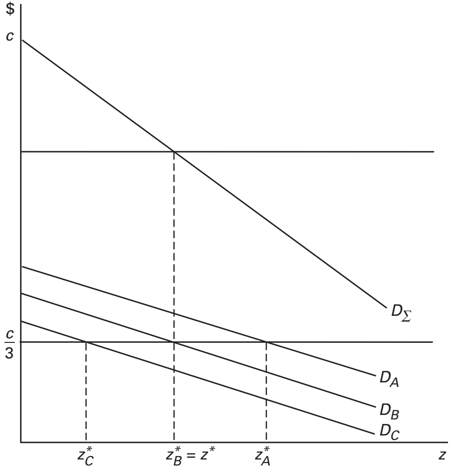
Sometimes various public goods are provided by local jurisdictions instead of by the state government. Such a good is called local public good . This in particular is the case in the United Stated of America and in Canada. Examples for such goods there include secondary education, police, fire protection and recreational facilities.

The theory for local public goods is more complicated as such a good leads to inherent dynamics effects: an explanation for this is that consumers pay a (uniform) tax to cover the cost of the public good and if they do not like the level of this tax, they move to another local jurisdiction. This is called voting with one's feet .

# Voting with one's feet

The model in the text book supposes that people pay uniform tax to cover the cost of a public good: if there are $n$ persons, then each person pays $cz/n$. This tax depends on the local jurisdiction. So in the above figure the cost per unit of $z$ for each person is $c/3$. So in fact a person's $X$ preferred level $z^{\star}_{X}$, lies at the intersection of the demand curve $D_X$ and the horizontal line $c/3$. See the next figure.

# Voting with one's feet (ctd.)




Persons prefer different levels of the public good. Voting processes may deal with this problem: candidates compete for votes by promising a particular level of the public good.

# Voting with one's feet (ctd.)

There is a whole theory for what is going on here. One important aspect in this context is the "median voter model". The Textbook further elaborates on this. (It is an accident related to the fact that the individual demand curves in this figure are evenly space, that in this figure $z_B^\star = z^\star$.) The Section "Hotelling Game" in Slides B relates to such a model.